Inequation:- The statement of inequality between two algebraic expressions is called an inequation.

Eg. (i) $2x + \frac{1}{2} \geq +3$

(ii) $x + \frac{7}{3} < 6$. etc.

The general form of linear inequation is given by, (i) $ax+b>0$ (ii) $ax+b \leq 0$

(iii) $ax+b < 0$ (iv) $ax+b \geq 0$

where a and b are real numbers and $a \neq 0$.

Properties of Inequalities

(1) If $a > b \Rightarrow a+c > b+c$

If $a < b \Rightarrow a+c < b+c$

(2) If $a > b \Rightarrow a-c > b-c$

If $a < b \Rightarrow a-c < b-c$

(3) If $a > b \Rightarrow a \times c > b \times c$, $c > 0$.

If $a \not< b \Rightarrow a \times c < b \times c$, $c > 0$

*(4) If $a > b \Rightarrow a \times c < b \times c$, if $c < 0$

If $a < b \Rightarrow a \times c > a \times c$, if $c < 0$.

(5) If $a > b \Rightarrow \frac{a}{c} > \frac{b}{c}$, if $c > 0$

If $a < b \Rightarrow \frac{a}{c} < \frac{b}{c}$, if $c > 0$

*(6) If $a > b \Rightarrow \frac{a}{c} < \frac{b}{c}$, if $c < 0$

If $a \not< b \Rightarrow \frac{a}{c} > \frac{b}{c}$, if $c < 0$

Exercise - 16A

1. (i) given, $x \in \{-3, -2, -1, 0, 1, 2, 3\}$

Now, $x + 2 < 1$

$\Rightarrow x < 1 - 2$

$\Rightarrow x < -1$

$\therefore$ solution set $= \{-3, -2\}$

(ii) $x \in \{-3, -2, -1, 0, 1, 2, 3\}$

$2x - 1 < 4$

$\Rightarrow 2x < 4 + 1$

$\Rightarrow 2x < 5$

$\Rightarrow x < \frac{5}{2}$ $\Rightarrow x < 2\frac{1}{2}$.

$\therefore$ soln set $= \{-3, -2, -1, 0, 1, 2\}$.

(iii) $\frac{2}{3}x < 1$, $x \in \{-3, -2, -1, 0, 1, 2, 3\}$

$\Rightarrow 2x < 3$

$\Rightarrow x < \frac{3}{2}$ $\Rightarrow x < 1\frac{1}{2}$.

$\therefore$ soln set $= \{-3, -2, -1, 0, 1\}$.

(iv) $1 - x > 0$, $x \in \{-3, -2, -1, 0, 1, 2, 3\}$

$\Rightarrow -x > -1$

$\Rightarrow x < 1$

$\therefore$ soln set $= \{-3, -2, -1, 0\}$.

(2)

(vi) $3-5x < -1$, $x \in \{-3,-2,-1,0,1,2,3\}$

$\Rightarrow -5x < -1-3$

$\Rightarrow -5x < -4$

$\Rightarrow 5x > 4 \Rightarrow x > \frac{4}{5}$

$\therefore$ soln set $= \{-3,-2,-1,0\}$

(vii) $2-3x > 1$, $x \in \{-3,-2,-1,0,1,2,3\}$

$\Rightarrow -3x > 1-2$

$\Rightarrow -3x > -1$

$\Rightarrow 3x < 1 \Rightarrow x < 1/3$

$\therefore$ soln set $= \{-3,-2,-1,0\}$

(viii) $-6 \geq 2x-4$

$\Rightarrow -6+4 \geq 2x$

$\Rightarrow -2 \geq 2x$

$\Rightarrow 2x \leq -2$

$\Rightarrow x \leq -1$

$\therefore$ soln set $= \{-3,-2,-1\}$

(viii) $3x-5 \geq -12$, $x \in \{-3,-2,-1,0,1,2,3\}$

$\Rightarrow 3x \geq -12+5$

$\Rightarrow 3x \geq -7$

$\Rightarrow x \geq -\frac{7}{3} \Rightarrow x \geq -2\frac{1}{3}$

$\therefore$ soln set $= \{-2,-1,0,1,2,3\}$

ix) $14-2x<6$, $x \in \{-3,-2,-1,0,1,2,3\}$

$\Rightarrow -2x \leq 6-14$

$\Rightarrow -2x \leq -8$

$\Rightarrow 2x \geq 8 \Rightarrow x \geq 4$

sol^n set $= \phi$.

Q.2. ~~(i) $9x<7 \leq 25+3x$, $x \in N$~~

(i) $3x-8<0$, $x \in N$

$\Rightarrow 3x<8$

$\Rightarrow x<8/3 \Rightarrow x<2\frac{2}{3}$.

$\therefore$ sol^n set $=\{1,2\}$

(ii) $7x+3 \leq 17$, $x \in N$

$\Rightarrow x \in \{1,2,3,\ldots\}$

$\Rightarrow 7x \leq 17-3$

$\Rightarrow 7x \leq 14$

$\Rightarrow x \leq 2$

$\therefore$ sol^n set $=\{1,2\}$

(iii) $5-x>1$, $x \in N$

$\Rightarrow x \in \{1,2,3,\ldots\}$

$\Rightarrow -x>1-5$

$\Rightarrow -x>-4$

$\Rightarrow x<4$

$\therefore$ sol^n set $=\{1,2,3\}$.

(5)

(iv) $1-3x > -4$, $x \in N$

$\Rightarrow -3x > -4-1$ $\Rightarrow x \in \{1, 2, 3, 4, \dots\}$

$\Rightarrow -3x > -5$

$\Rightarrow 3x < 5 \Rightarrow x < 5/3 \Rightarrow x < 1\frac{2}{3}$.

$\therefore$ soln set $= \{1\}$.

(v) $\frac{3}{2} - \frac{x}{2} > -1$, $x \in N \Rightarrow x \in \{1, 2, 3, 4, \dots\}$

$\Rightarrow \frac{3-x}{2} > -1$

$\Rightarrow 3-x > -2$.

$\Rightarrow -x > -5 \Rightarrow x < 5$.

$\therefore$ soln set $= \{1, 2, 3, 4\}$.

(vi) $-\frac{1}{4} \leq \frac{1}{2} - \frac{x}{3}$, $x \in N$

$\Rightarrow x \in \{1, 2, 3, \dots\}$

$\Rightarrow -\frac{1}{4} - \frac{1}{2} \leq -\frac{x}{3}$.

$\Rightarrow \frac{-1-2}{4} \leq \frac{-x}{3}$.

$\Rightarrow \frac{-3}{4} \leq -\frac{x}{3}$.

$\Rightarrow -9 \leq -4x$

$\Rightarrow 9 \geq 4x \Rightarrow \frac{9}{4} \geq x \Rightarrow x \leq 2\frac{1}{4}$.

$\therefore$ soln set $= \{1, 2\}$.

Q.3. (i) $9x - 7 \leq 25 + 3x$, [$x \in Z$. $\Rightarrow x \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$]

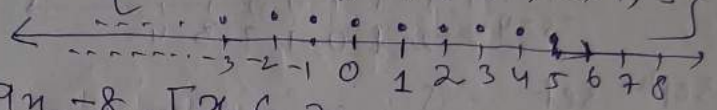
$\Rightarrow 9x - 3x \leq 25 + 7$.

$\Rightarrow 6x \leq 32$

$\Rightarrow x \leq \frac{32}{6}$ 16

$\Rightarrow x \leq 5\frac{1}{3}$.

$\therefore$ soln set $= \{\ldots, -2, -1, 0, 1, 2, 3, 4, 5\}$



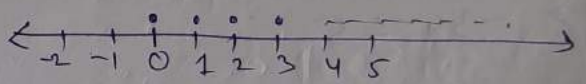
(ii) $-17 < 9x - 8$, [$x \in Z$ $\Rightarrow x \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$]

$\Rightarrow -17 + 8 < 9x$

$\Rightarrow -9 < 9x$.

$\Rightarrow 9x > -9$

$\Rightarrow x > -1$



$\therefore$ soln set $= \{0, 1, 2, \ldots\}$.

(iii) $-4(x+5) > 10$, [$x \in Z$ $\Rightarrow x \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$]

$\Rightarrow -4x - 20 > 10$

$\Rightarrow -4x > 30$

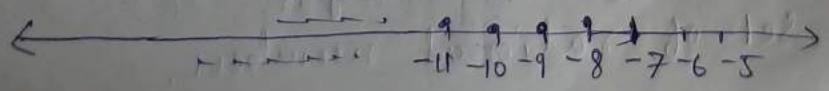
$\Rightarrow 4x < -30$

$\Rightarrow x < \frac{-30}{4}$.

$\Rightarrow x < -7\frac{2}{4}$.

$\Rightarrow x < -7\frac{1}{2}$

$\therefore$ soln set $= \{\ldots, -9, -8\}$.



(iv) $4 - 3x < 13 + x$, $x \in Z$

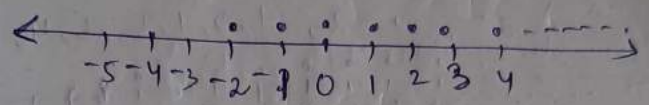
$\Rightarrow x \in \{\dots, -2, -1, 0, 1, 2, \dots\}$

$\Rightarrow -3x - x < 13 - 4$

$\Rightarrow -4x < 9$

$\Rightarrow 4x > -9$

$\Rightarrow x > \frac{-9}{4} \Rightarrow x > -2\frac{1}{4}$.



soln set $= \{-2, -1, 0, 1, 2, 3, \dots\}$

(v) $5 - 4x < 10 - x$, $x \in Z$

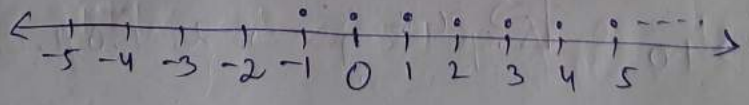
$\Rightarrow x \in \{\dots, -2, -1, 0, 1, 2, \dots\}$

$\Rightarrow -4x + x < 10 - 5$

$\Rightarrow -3x < 5$

$\Rightarrow 3x > -5$

$\Rightarrow x > \frac{-5}{3} \Rightarrow x > -1\frac{2}{3}$.



$\therefore$ soln set $= \{-1, 0, 1, 2, \dots\}$

Notations for different sets

N = set of natural numbers.
W = set of whole numbers.
Z = set of Integers.
Z^+ = set of +ve integers
Z^- = set of -ve integers
Q = set of rational numbers.

(iv) $10 - 2(1+4x) < 20$ [$x \in Z$. $\Rightarrow x \in \{\dots, -1, 0, 1, \dots\}$]

$\Rightarrow 10 - 2 - 8x < 20$.

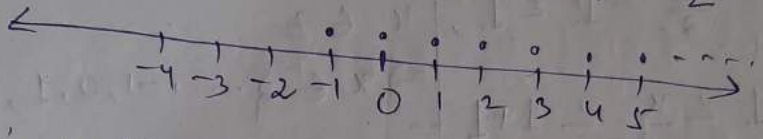
$\Rightarrow -8x < 20 - 10 + 2$.

$\Rightarrow -8x < 12$.

$\Rightarrow 8x > -12$

$\Rightarrow x > -\frac{12}{8}^{3}_{2}$ $\Rightarrow x > -1\frac{1}{2}$

$\Rightarrow x > -1\frac{1}{2}$



$\therefore$ soln set $= \{-1, 0, 1, 2, 3, \dots\}$

4. (i) $1 - 4x \geq -1$, [$x \in N$ $\Rightarrow x \in \{1, 2, 3, 4, \dots\}$]

$\Rightarrow -4x \geq -1 - 1$

$\Rightarrow -4x \geq -2$.

$\Rightarrow 4x \leq 2 \Rightarrow x \leq 1/2$.

$\therefore$ soln set $= \phi$.

(ii) $-3 \leq 4x + 1 < 9$, [$x \in Z$ $\Rightarrow x \in \{\dots, -1, 0, 1, \dots\}$]

$\Rightarrow -3 + 1 \leq 4x + 1 - 1 < 9 - 1$

$\Rightarrow -4 \leq 4x < 8$.

$\Rightarrow \frac{-4}{4} \leq \frac{4x}{4} < \frac{8}{4}$.

$\Rightarrow -1 \leq x < 2$.

$\therefore$ soln set $= \{-1, 0, 1\}$

(2)

(iii) $0 < 2x - 5 < 5$, $\left[\begin{array}{l} x \in W \\ \Rightarrow x \in \{0, 1, 2, 3, \cdots\} \end{array}\right.$

$\Rightarrow 5 < 2x - 5 + 5 < 5 + 5$

$\Rightarrow 5 < 2x < 10$

$\Rightarrow \frac{5}{2} < x < 5 \Rightarrow 2\frac{1}{2} < x < 5$

$\therefore$ soln set $= \{3, 4\}$.

(iv) $-3 < \frac{x}{2} - 1 < 1$, $\left[\begin{array}{l} x \in Z \\ \Rightarrow x \in \{\cdots, -1, 0, 1, \cdots\} \end{array}\right.$

$\Rightarrow -3 + 1 < \frac{x}{2} < 1 + 1$

$\Rightarrow -2 < \frac{x}{2} < 2$.

$\Rightarrow -4 < x < 4$.

$\therefore$ soln set $= \{-3, -2, -1, 0, 1, 2, 3\}$.

(v) $-4 < \frac{2x}{5} + 1 < -3$, $\left[\begin{array}{l} x \in Z \\ \Rightarrow x \in \{\cdots, -1, 0, 1, \cdots\} \end{array}\right.$

$\Rightarrow -4 + 1 < \frac{2x}{5} < -3 - 1$

$\Rightarrow -5 < \frac{2x}{5} < -4$

$\Rightarrow -25 < 2x < -20$.

$\Rightarrow -\frac{25}{2} < x < -\frac{20}{2}$.

$\Rightarrow -12\frac{1}{2} < x < -10$

$\therefore$ soln set $= \{-12, -11\}$

(vi) $-1 < \frac{2x}{3} + 1 \leq 5$, [$x \in Q$. where, Q is the set of rational nos.

$\Rightarrow -1-1 < \frac{2x}{3} \leq 5-1$

$\Rightarrow -2 < \frac{2x}{3} \leq 4$

$\Rightarrow -6 < 2x \leq 12$.

$\Rightarrow \frac{-6}{2} < x \leq \frac{12}{2}$.

$\Rightarrow -3 < x \leq 6$.

$\therefore$ soln set is $\{x \in Q : -3 < x \leq 6\}$.

(vii) $-\frac{1}{4} \leq \frac{1}{2} - \frac{x}{3} < 2$, [$x \in Z$ $\Rightarrow x \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

~~x~~.

$\Rightarrow -\frac{1}{4} - \frac{1}{2} \leq -\frac{x}{3} < 2 - \frac{1}{2}$.

$\Rightarrow \frac{-3}{4} \leq \frac{-x}{3} < \frac{3}{2}$

$\Rightarrow -\frac{9}{4} \leq -x < \frac{9}{2}$.

$\Rightarrow \frac{9}{4} \geq x > -\frac{9}{2}$

$\Rightarrow -\frac{9}{2} < x \leq \frac{9}{4}$.

$\therefore \Rightarrow -4\frac{1}{2} < x \leq 2\frac{1}{4}$

$\therefore$ soln set $= \{-4, -3, -2, -1, 0, 1, 2\}$

(viii) $3 + \frac{x}{4} < \frac{2x}{3} + 5$, $x \in Q$,
Q is the set of rational nos.

$\Rightarrow \frac{x}{4} - \frac{2x}{3} < 5 - 3$

$\Rightarrow \frac{3x - 8x}{12} < 2.$

$\Rightarrow -5x < 24.$

$\Rightarrow 5x > -24.$

$\Rightarrow x > -\frac{24}{5} \Rightarrow x > -4\frac{4}{5}$

$\therefore$ soln set $= \{ x \in Q : x > -4\frac{4}{5} \}$

(ix) $\frac{3x+1}{4} \leq \frac{5x-2}{3}$, $x \in Q$.

$\Rightarrow 3(3x+1) \leq 4(5x-2)$

$\Rightarrow 9x + 3 \leq 20x - 8$

$\Rightarrow 9x - 20x \leq -8 - 3$

$\Rightarrow -11x \leq -11$

$\Rightarrow 11x \geq 11 \Rightarrow x \geq 1.$

$\therefore$ soln set $= \{ x \in Q : x \geq 1 \}$

(x) $\frac{1}{3}(4x-1)+3 \leq 4+\frac{2}{5}(6x+2)$, $x \in Q$.

$\Rightarrow \frac{4x-1+9}{3} \leq \frac{20+12x+4}{5}$.

$\Rightarrow \frac{4x+8}{3} \leq \frac{12x+24}{5}$.

$\Rightarrow 20x+40 \leq 36x+72$

$\Rightarrow 20x-36x \leq 72-40$

$\Rightarrow -16x \leq 32$

$\Rightarrow 16x \geq -32 \Rightarrow x \geq -2$.

$\therefore$ soln set $= \{x \in Q : x \geq -2\}$.